Math 333 Homework Problems #6

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

4.3. Laplace's equation

- 4.3.3
- 4.3.9 Consider the *exterior* boundary value problem

$$\begin{split} \Delta u &= 0, \quad r > R, \ 0 \leq \theta < 2\pi \\ u(R,\theta) &= f(\theta), \ 0 \leq \theta < 2\pi. \end{split}$$

Find the series solution. (Note that in Problem 4.3.1 the series solution can be represented as an integral.)

• 4.3.10 This is an extension of Problem 4.3.3. Solve the following boundary value problem on a wedge:

$$\begin{split} \Delta u &= 0, \quad 0 < r < R, \; 0 \leq \theta \leq \pi/2 \\ u(R,\theta) &= f_{\rm R}(\theta), \; 0 \leq \theta \leq \pi/2 \\ u(r,0) &= 0, \; u_{\theta}(r,\pi/2) = f_{\rm T}(r), \; 0 < r < R. \end{split}$$

Plot the solution for the special case of

$$R = 4$$
, $f_{\rm R}(\theta) = \sin(\theta) - .7\sin(3\theta)$, $f_{\rm T}(r) = r(R/3 - r)(R - r)$.