MATH W81: HOMEWORK #2

1. Let $\boldsymbol{A} \in \mathbb{C}^{n\times n}$, i.e., $\boldsymbol{A}$ is a $n \times n$ matrix with complex-valued coefficients. Show that

$$\langle \boldsymbol{Ax}, \boldsymbol{y} \rangle = \langle \boldsymbol{x}, \boldsymbol{A}^{\mathrm{H}} \boldsymbol{y} \rangle.$$

2. Find an orthonormal basis for

$$\mathrm{Span}\{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \\ 0 \end{pmatrix}\}.$$

3. Suppose that

$$S = \mathrm{Span}\{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \\ -1 \end{pmatrix}\}.$$

(a) Find an orthonormal basis for S.

(b) Find an orthonormal basis for $S^{\perp}$.