

Abstracts

of talks presented at the
19th Annual Workshop in Geometric Topology
June 13–15, 2002

Distinguishing Croke-Kleiner Boundaries

Ric Ancel

University of Wisconsin at Milwaukee

Abstract. (Joint work with Julia Wilson.) The Croke-Kleiner spaces $X(a)$, $0 < a \leq \pi/2$, are CAT(0) 2-complexes that are all acted upon geometrically by the same group. In their article in *Topology* 39 (2000), C. Croke and B. Kleiner proved that for $a < \pi/2$, $X(a)$ and $X(\pi/2)$ have non-homeomorphic visual boundaries. We will sketch a proof that if $a < \pi/2n \leq b$ for some integer $n \geq 1$, then $X(a)$ and $X(b)$ have non-homeomorphic visual boundaries. (J. Wilson's thesis under F. Ancel's supervision contained a flawed proof that if $a < b$, then $X(a)$ and $X(b)$ have non-homeomorphic visual boundaries. This assertion is currently unproved.)

A Description of Ribbon Disk Complements

Tony Bedenikovic

Bradley University

Abstract. In this talk I wish to describe ribbon disk complements, which are the complements of particular 2-dimensional disks in the 4-ball. The description here will be highly visual. (Other, more algebraic descriptions are available.) We'll see that ribbon disk complements generalize the notion of knot complements in the 3-sphere. Many of the questions asked of them, therefore, are similar to those asked of knot complements. I will offer a pleasant, though impractical condition which implies the asphericity of ribbon disk complements.

Property A for groups acting on metric spaces

Greg Bell

University of Florida

Abstract. Yu introduced property A for groups and discrete metric spaces. A finitely generated group with property A embeds uniformly into Hilbert space and thus, the Novikov Higher Signature Conjecture holds for that group. We prove that groups acting by isometries on metric spaces with finite asymptotic dimension will have property A provided neighborhoods of the stabilizers of the action have property A. In particular we conclude a result of Tu according to which amalgamated products and HNN extensions of groups with property A will also have property A.

Multiply connected models in cosmology

Lawrence Brenton
Wayne State University

Abstract. The past decade has seen an explosion of interest among astronomers and cosmologists in the possibility that the universe is not simply connected, with many observational experiments under way or in prospect for the detection of topological structure (so far without success). In this talk I will present a classification of certain closed "almost Robertson-Walker" models in which the assumption of simple connectivity is replaced by the weaker condition that the spatial cross-sections are homology three-spheres. I will also discuss a new space-time model which is topologically the cone on a twisted circle-bundle on the two-torus, and derive some of its physical properties as exhibited by the Einstein field equations.

Global sections in Serre fibration with fibers homeomorphic to 3-manifold

Nikolay Brodsky
University of Saskatchewan

Abstract. We prove the following theorem on sections of fibration over an infinite-dimensional base.

Theorem. *Let $p : E \rightarrow B$ be topologically regular mapping of compacta with fibers homeomorphic to some compact connected 3-dimensional manifold M . If B is ANR-space, then p admits a global section if either of the following conditions hold:*

- (a) $\pi_1(M)$ is abelian, M is aspheric, and $H^2(B; \pi_1(F_b)) = 0$;
- (b) M is closed hyperbolic 3-manifold and $\pi_1(B) = 0$;
- (c) M is closed, irreducible, sufficiently large, contains no embedded $\mathbb{R}P^2$ having a trivial normal bundle, and $\pi_1(B) = \pi_2(B) = 0$.

Note that if the Poincaré Conjecture is true, then any Serre fibration of LC^2 -compacta with fibers homeomorphic to 3-manifold is topologically regular.

Reflections on the Bing-Borsuk Conjecture

John Bryant
Florida State University

Abstract. In their paper, Some remarks concerning topologically homogeneous spaces, *Ann. of Math.* **81** (2) (1965), 100 - 111, Bing and Borsuk proved that a homogeneous ENR of dimension 1 or 2 is a topological manifold. Implicit in their paper is the conjecture that the same should be true in every dimension. In light of the many exotic examples in higher dimensions, particularly the examples of Bryant-Ferry-Mio-Weinberger, a more reasonable conjecture is that a homogeneous ENR is a homology manifold. If the local homology groups of a homogeneous ENR X are finitely generated, then it is known that X is a homology manifold, but attempts to remove this assumption have proved unsuccessful

so far. We will discuss sheaf-theoretic techniques that have been used to attack this problem and the extent to which they have been found wanting. We offer some conjectures reminiscent of Borsuk's "umbrella theorem" that could lead to a solution.

Constructing 3-Manifolds

James Cannon
Brigham Young University

Abstract. Face-pairings, Heegaard splittings, surgery descriptions, and their relationships, with special attention to twisted face-pairing 3-manifolds

On certain i -d compacta and an application

Tadek Dobrowolski
Pittsburg State University

Abstract. A typical property for an infinite-dimensional compactum K is that the square of K is homeomorphic to K . We will discuss a few properties that are stronger than the NEGATION of this typical property. An application to absorbing sets will follow.

Dimension theory: local and global

Alexander Dranishnikov
University of Florida

Abstract. I will speak about the Alexandroff Problem, both local and global, and its relation to geometric topology and in particular to the Novikov Higher Signature Conjecture.

The algebra of dimension theory

Jerzy Dydak
University of Tennessee

Abstract. We present an approach to cohomological dimension theory based on infinite symmetric products and on the general theory of dimension called the extension dimension. The notion of the extension dimension $e - \dim(X)$ was introduced by A. N. Dranishnikov in the context of compact spaces and CW complexes. This paper investigates extension types of infinite symmetric products $SP^\infty(L)$. One of the main ideas of the paper is to treat $e - \dim(X) \leq SP^\infty(L)$ as the fundamental concept of cohomological dimension theory instead of $\dim_G(X) \leq n$. Properties of infinite symmetric products lead naturally to a calculus of graded groups which implies most of classical results of the cohomological dimension. The basic notion is that of homological dimension of a graded group which allows for simultaneous treatment of cohomological dimension of compacta and extension

properties of CW complexes. We introduce cohomology of X with respect to L (defined as homotopy groups of the function space $SP^\infty(L)^X$). Another main idea is to treat homology and cohomology on the same level which allows introduction of the dual graded group as an algebraic analog of Dranishnikov Duality.

As an application of our results we characterize all countable groups G so that the Moore space $M(G, n)$ is of the same extension type as the Eilenberg-MacLane space $K(G, n)$. Another application is characterization of infinite symmetric products of the same extension type as a compact (or finite-dimensional and countable) CW complex.

A complete group of wild braids

Paul Fabel

Mississippi State University

Abstract. Artin's braid group B_n is the fundamental group of Y^n , the collection of subsets of the plane with exactly n elements. Choosing nested basepoints $A_n \in Y_n$, Artin's braid group on infinitely many strands B_∞ is the direct limit of B_n via monomorphisms $i_n : B_n \rightarrow B_{n+1}$ that attach a 'trivial' strand to the braid $b_n \in \pi_1(Y^n, A_n)$.

We construct a completely metrizable topological group of wild braids $\overline{B_\infty}$ in which B_∞ is a densely embedded subgroup.

Geometrically each element of $\overline{B_\infty}$ is represented as a subspace of $R^2 \times [0, 1]$ consisting of countably many arcs connecting $Z^+ \times \{0\}$ to $Z^+ \times \{1\}$.

Embedding homology manifolds in codimension

Steve Ferry

Rutgers University

Abstract. We prove that every exotic homology n -manifold, $n > 5$, homotopy equivalent to the sphere embeds in S^{n+2} . This is joint work with Shmuel Weinberger.

On homotopy properties of certain Coxeter group boundaries

Hanspeter Fischer

Ball State University

Abstract. (Joint work with Craig Guilbault, University of Wisconsin-Milwaukee.) There is a natural homomorphism from the fundamental group of the visual boundary of any non-positively curved geodesic space to its fundamental group at infinity. In this CAT(0) setting, the latter group coincides with the first shape group of the visual boundary. This talk will focus on certain Coxeter group boundaries and closely related spaces, where the abovementioned homomorphism turns out to be injective.

Taming Manifolds that are Wild at Infinity

Craig R. Guilbault

University of Wisconsin Milwaukee

Abstract. One of the best known and most frequently applied theorems in the study of non-compact manifolds is found in L. C. Siebenmann’s 1965 Ph.D. thesis. It gives necessary and sufficient conditions for the end of an open manifold to possess the simplest possible structure—that of an open collar.

Theorem (Siebenmann). *A one ended open n -manifold M^n ($n \geq 6$) contains an open collar neighborhood of infinity iff each of the following is satisfied:*

- (1) M^n is inward tame at infinity,
- (2) $\pi_1(\varepsilon(M^n))$ is stable, and
- (3) $\sigma_\infty(M^n) \in \tilde{K}_0(\mathbb{Z}[\pi_1(\varepsilon(M^n))])$ is trivial.

One of the beauties of Siebenmann’s theorem is the simple structure it places on the ends of certain manifolds. At the same time, this simplicity greatly limits the class of manifolds to which the theorem applies. Indeed, many interesting and important non-compact manifolds are “too complicated at infinity” to be collarable. Frequently the condition these manifolds violate is π_1 -stability. In this talk we will discuss an ongoing program to obtain variations on this theorem which apply to manifolds with non-stable fundamental groups at infinity.

The main focus will be on a recently discovered example (joint with Fred Tinsley) that illustrates the appropriate replacement for Condition (2).

Trees, ultrametrics and C^* -algebras

Bruce Hughes

Vanderbilt University

Abstract. I will discuss how ideas in noncommutative geometry can be used to study the micro-geometry of ultrametric spaces. In particular, I will introduce a C^* -algebra which is a local similarity invariant of ultrametric spaces. Via end theory, this leads to a large-scale isometry invariant for real trees.

Centralizers in the braid groups

Nikolai V. Ivanov

Michigan State University

Abstract. We will outline a counterexample to a recent conjecture of N. Franco and J. Gonzales-Meneses about number of generators in the centralizers of elements in the Artin braid groups. An interesting feature of this result is the fact that the conjecture is well supported by an algebraic evidence, yet the topological considerations lead to a rather transparent disproof. Some positive results also will be discussed.

Efficient triangulations of 3-manifolds (with applications)

William Jaco
Oklahoma State University

Abstract. We introduce the concepts of “0- and 1-efficient triangulations” of 3-manifolds. While efficient triangulations severely limit the existence of normal 2-spheres and disks (0-efficient) and normal tori (1-efficient), they exist quite generally and are useful in both understanding 3-manifolds and in applications. Except for the 3-sphere, these triangulations have only one-vertex and exhibit special local features. They serve as useful tools in computation and are used to complete the proof of Waldhausen’s Conjecture that closed 3-manifolds only have a finite number of Heegaard splittings of a bounded genus (up to homeomorphism and isotopy). They also provide new proofs of unique irreducible Heegaard splittings of the 3-sphere and lens spaces, as well as a classification of Heegaard splittings for Seifert fibered manifolds and certain surface bundles.

The coarse Borel conjecture, bounded surgery groups and non-uniformly contractible spaces

Heather Johnston
Vassar College

Abstract. The Borel conjecture states that if BG is a finite Poincaré complex, then it has a unique closed compact topological manifold in its homotopy type. In many cases, this is equivalent to the existence of a topological manifold surgery exact sequence for BG in which the assembly map is an isomorphism. An analogous coarse Borel conjecture about coarse homology and bounded surgery obstruction groups of EG implies the Novikov conjecture for G . We reformulate and test a version of the coarse Borel conjecture for non-uniformly contractible spaces.

On strictly contractible polyhedra

Umed H. Karimov
Academy of Sciences, Tajikistan

Abstract. Michael introduced and studied the conception of strict contractibility [M]. He stated some interesting questions on which Dydak, Segal and Spiez printed the answer [DSS]. But there is a gap in their statement on our opinion. The purpose of our note is to fill this gap and discuss some problems on strict contractibility.

This is part of joint work with D. Repovš on strict contractibility.

Definition. A homotopy $H : X \times I \rightarrow X$ (I is the segment $[0, 1]$) is called a *contraction to a point* $x_0 \in X$ if for every $x \in X$, $H(x, 0) = x$ and $H(x, 1) = x_0$.

Definition. The space X is called *simple contractible* to a point $x_0 \in X$ if there exists a contraction $H : X \times I \rightarrow X$ to the point x_0 such that:

- (a) $H(x, t) = x_0$ implies $x = x_0$ or $t = 1$,

(b) $H(x_0, t) = x_0$ for all t .

A space X is called *strictly contractible* to $x_0 \in X$ if for the contraction only condition (a) holds [M].

Our definition of simple contractibility at bottom is the same as definition of strictly contractibility by Dydak, Segal and Spiez (cf. [DSS] p.1).

Obviously every simply contractible space is strictly contractible but the compactum $E = \{1, \frac{1}{2}, \frac{1}{3}, \dots, 0\} \times I \cup I \times \{0\}$ (the Comb Space, see e.g. Example 1.4.8 in [Sp]) is strictly contractible not simply contractible to the point $(0, 1) \in E$ space.

So the definition of strictly contractibility of Dydak, Segal and Spiez are different from Michael one. They proved that there exist contractible polyhedrons X which are not simple contractible to some points $x_0 \in X$ and $X \setminus \{x_0\}$ are ARs.

We prove applying the method of the paper [KR] that contractible polyhedrons constructed by Dydak, Segal and Spiez are not strictly contractible. This is answer to the questions of Michael ([DSS], [M]).

References.

[DSS] J. Dydak, J. Segal and S. Spiez, *On question of strict contractibility*, Topol. Appl., to appear.

[KR] U. H. Karimov and D. Repovš, *On suspensions of noncontractible compactum of triall shape*, Proc. Amer. Math. Soc. 127 (1999), 627-632.

[M] E. Michael, *Closed retracts and perfect retracts*, preprint, Univ. of Washington, Seattle, 1999.

[Sp] E. H. Spanier, *Algebraic Topology*, Springer-Verlag, New York, 1966.

On embedding of compacta into the product of curves

Akira Koyama

Osaka Kyoiku University, Japan

Abstract. We are introducing a criterion of n -dimensional compacta which cannot be embedded into the product of n curves. We are generalizing the idea to an unembedding property of 2-dimensional compacta into the second symmetric product of curves.

Unique factorization of graph products of groups

David Radcliffe

University of Wisconsin-Milwaukee

Abstract. We show that if a group can be represented as a graph product of finite indecomposable groups then this representation is unique.

***G*-acyclic Resolutions for Metrizable Compacta**

Leonard R. Rubin and Philip J. Schapiro
University of Oklahoma and Langston University

Abstract. We shall prove a *G*-acyclic resolution theorem for \dim_G , cohomological dimension modulo an arbitrary abelian group *G*, in the class of metrizable compacta. This means that, given a metrizable compactum *X* such that $\dim_G X \leq n$ ($n \geq 2$), there exists a metrizable compactum *Z* and a surjective map $\pi : Z \rightarrow X$ such that:

- (a) π is *G*-acyclic,
- (b) $\dim Z \leq n + 1$, and
- (c) $\dim_G Z \leq n$.

To say that a map π is *G*-acyclic, for an abelian group *G*, means that each fiber $\pi^{-1}(x)$ of π is *G*-acyclic, i.e., that all the reduced Čech cohomology groups of $\pi^{-1}(x)$ modulo the group *G* are trivial.

Homogeneous *C-algebras**

Mikhail Shchukin
Belarusian State University

Abstract. Every homogeneous *C**-algebra corresponds to an algebraic fibre bundle. A *C**-algebra is called *non-trivial* if the corresponding algebraic fibre bundle is non-trivial. All *C**-algebras generated by idempotents that were studied before corresponded to trivial algebraic fibre bundles. In this work it is shown that every homogeneous separable non-commutative *C**-algebra can be generated by idempotents. It follows that we need to study the topological properties of *C**-algebras generated by idempotents to describe such algebras properly. Also in the work we found the minimal number of idempotent generators for every homogeneous *C**-algebra *A* with the set of maximal ideals homeomorphic to the sphere S^2 .

Perfect subgroups of HNN extensions

F. C. Tinsley
Colorado College

Abstract. (Joint work with Craig R. Guilbault) The simplest Baumslag-Solitar group has the presentation $\langle y_1, y_0 | y_1^2 = y_0^{-1} y_1 y_0 \rangle$. This group has no non-trivial perfect subgroups primarily because its commutator subgroup is abelian. We use basic subgroup theorems for HNN extensions to show that the generalized Baumslag-Solitar group,

$$G_K = \langle y_0, y_1, \dots, y_k | y_1^2 = y_0^{-1} y_1 y_0, y_2^2 = y_1^{-1} y_2 y_1, \dots, y_k^2 = y_{k-1}^{-1} y_k y_{k-1} \rangle$$

has no non-trivial perfect subgroups. We use these groups to construct a manifold with a strange tame end.

We contrast our construction to one using Adam's group,

$$\left\langle y_1, y_0 \mid y_1^2 = y_1^{(y_1^{y_0})} \right\rangle$$

whose commutator subgroup is, indeed, perfect.

**When is a manifold triangulated?
Revisiting a classical result from the viewpoint of homotopy theory**

James Turner
Calvin College

Abstract. The question of whether an n -manifold is triangulated (or smooth, or has whatever favorite structure you wish to impose upon it) has been decisively answered through the work of Kirby and others. In this talk, we describe how their approach can be recast in terms of the more recent homotopy theory of moduli spaces of structures and how their obstruction groups fall out easily from this description (though not their computation).

Homeomorphisms of the Whitehead manifold

David Wright
Brigham Young University

Abstract. (Joint work with Kathy Andrist.) We show that there are no orientation reversing homeomorphisms of the Whitehead manifold.