Finite dimensionality of $\mathcal{Z}$-boundaries and its consequences

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Abstract. The rich study of boundaries of CAT(0) and hyperbolic groups led M. Bestvina to formalize the concept of a group boundary by defining a $\mathcal{Z}$-structure on a group. In his original definition, a $\mathcal{Z}$-structure on a group G is a pair of spaces $(\hat{X}, Z)$ where $\hat{X}$ is a compact ER, Z is a $\mathcal{Z}$-set in $\hat{X}$, G acts freely, cocompactly, and properly on $X = \hat{X} - Z$ and the collection of G-translates of a compact set in X forms a null sequence in $\hat{X}$. In this setting, Z is finite dimensional. We show that this result can be extended to the case that $\hat{X}$ is an AR, that is when $\hat{X}$ need not be finite dimensional. We also explore results that can be obtained by knowing the $\mathcal{Z}$-boundary is finite dimensional.